

Transport signatures of Floquet Majorana fermions in driven topological superconductors

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Floquet Majorana fermions are steady states of equal superposition of electrons and holes in a periodically driven superconductor. We study the experimental signatures of Floquet Majorana fermions in transport measurements and show, both analytically and numerically, that their presence is signaled by a quantized conductance sum rule over discrete values of lead bias differing by multiple absorption or emission energies at drive frequency. We also study the effects of static disorder and find that the quantized sum rule is robust against weak disorder. Thus, we offer a unique way to identify the topological signatures of Floquet Majorana fermions.

Introduction.—The nonlocal quantum order characterizing the topological state of a gapped medium often necessitates the existence of topologically protected gapless states bound to bulk defects or the edge with a topologically trivial medium where the gap closes. The detection of these topological bound states is, therefore, a primary probe of the topological state. It has recently been understood that topological bound states may arise as steady states when a topologically trivial system is driven periodically [1, 2]. In the superconducting state, these are equal superpositions of electrons and holes known as Floquet Majorana fermions [3, 4] that exhibit non-Abelian statistics [5, 6] and can be used for topological quantum computation [7]. This possibility expands the systems and conditions that realize Majorana fermions as emergent quasiparticles [8–16], but also poses fundamental questions as to how to detect and possibly manipulate such steady states. In particular, since the driven system is not in equilibrium, the experiments probing the equilibrium response of static Majorana fermions [17–20] cannot be used directly for this purpose.

In this paper, we address these questions by studying the non-equilibrium transport properties of Floquet Majorana fermions. We show, both analytically and numerically, that there is a quantized conductance sum rule, which we dub the “Floquet sum rule,” whenever Floquet Majorana fermions exist. The Floquet sum rule naturally generalizes the quantized zero-bias conductance of static Majorana fermions [21–26]. Moreover, we show that the Floquet sum rule is robust against moderate static disorder, owing to its topological character, while other peaks get suppressed. Remarkably, this suggests that disorder, usually detrimental to electronic properties, can be used as a “sieve” to find Floquet Majorana fermions. Transport studies in irradiated graphene, where the Floquet topological insulator was first proposed to exist [1], suggested quantized transport in the driven system is possible in certain geometries and for large drive frequencies [36, 37]. We use a systematic Green’s function method that extends the previous studies to superconducting systems in any frequency range, and can, in principle, incorporate the effects of interactions.

Though our results are applicable to any realization of Floquet Majorana fermions, systems of cold atoms could prove specially useful in this regard due to a high degree of design control and newly developed experimental probes of their dynamics, such as single-atom imaging, tunneling, and transport [27–33]. Disorder can be introduced in cold atom systems controllably [34, 35] and could, therefore, play a key role in the detection and manipulation of Floquet Majorana fermions.

Model.—We study the model Hamiltonian $H(t) = H_w(t) + H_c + H_l$, where the last term describes the leads,

$$H_w(t) = \frac{i}{2} \gamma^\top A(t) \gamma, \quad (1)$$

is the Hamiltonian of the system (wire) in the Majorana basis $\gamma^\top = (\gamma_1, \dots, \gamma_{2N})$ with a real, skew-symmetric matrix $A(t)$, and the contact Hamiltonian $H_c = \sum_\lambda a^{\lambda\dagger} K^\lambda \gamma + \text{h.c.}$ with $a^{\lambda\dagger}$ is the row of electronic creation operators in lead λ , and K^λ a contact matrix.

Our analytical results are presented for a general realization of Majorana fermions, but we do have to pick a specific model for numerical calculations. Perhaps the simplest model system that realizes emergent Majorana fermions is a quantum wire with superconducting pairing in a spin-polarized electronic band [8, 38]. This simple model can be realized in solid state [13, 14, 17, 18] and potentially in cold atom systems [3, 39–42]. Multiple bands in the wire can be included in γ such that $N = N_b L$, where N_b is the number of bands and L is the number of sites. Each band furnishes two Majorana operators $(\gamma_{r1}, \gamma_{r2}) \equiv \gamma_r^\top$ at site r . The contact matrix is $K^\lambda \propto \langle a^\lambda c^\dagger \rangle \otimes (1, i)$ in the Majorana basis, where the second factor acts on the Majorana indices 1 and 2 at each site.

For the single band wire ($N_b = 1$) the only nonzero elements of A are

$$A_{r,r} = -i\mu_r\sigma_y, \quad A_{r,r+1} = \Delta_r\sigma_x + i w_r\sigma_y, \quad (2)$$

where the real parameters μ_r , Δ_r and w_r are, respectively, the chemical potential at site r , the superconducting pairing, and the hopping integral on $(r, r+1)$ bond, and $(\sigma_x, \sigma_y, \sigma_z)$ are Pauli matrices. The static,

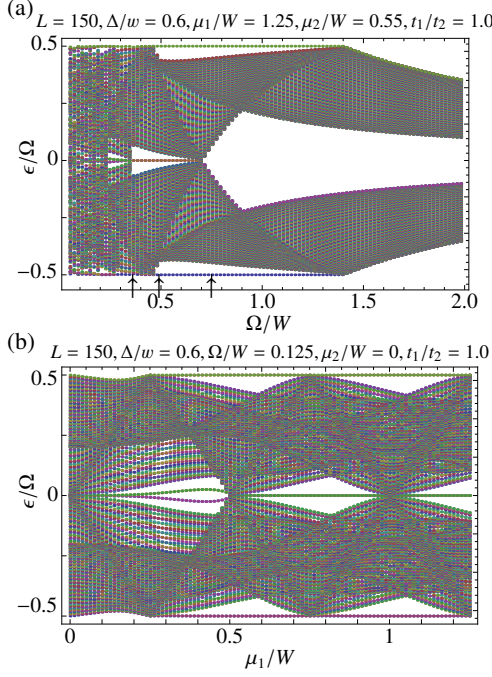


FIG. 1. The evolution of the Floquet spectrum of a quantum wire with L sites for a square-wave chemical potential, with (a) Ω/W and (b) μ_1/W . The arrows in (a) show the frequencies used for the transport calculations in Fig. 3.

uniform wire with bandwidth $W = 4|\text{Re } w|$ has a topological phase transition at $2|\mu| = W$ where the gap closes. There is an unpaired Majorana fermion at each end of the wire with zero energy (the energy is referenced to chemical potential of the wire) in the topological phase $2|\mu| < W$ and none in the trivial phase $2|\mu| > W$ [8].

When the system is driven with period $T \equiv 2\pi/\Omega$, the general solution of the time-periodic Schrödinger equation (we set $\hbar = 1$) $H(t)|\psi(t)\rangle = i\partial_t|\psi(t)\rangle$ is found in terms of the Floquet functions $|\psi_\alpha(t)\rangle = e^{-i\epsilon_\alpha t}|\phi_\alpha(t)\rangle$, where $|\phi_\alpha(t+T)\rangle = |\phi_\alpha(t)\rangle$ is an eigenket of the effective Hamiltonian $H_{\text{eff}}(t) = H(t) - i\partial_t$, $H_{\text{eff}}(t)|\phi_\alpha(t)\rangle = \epsilon_\alpha|\phi_\alpha(t)\rangle$. The quasienergies ϵ_α are restricted to $(-\Omega/2, \Omega/2]$ by the map $\epsilon_\alpha \mapsto \epsilon_\alpha + k\Omega$, $|\phi_\alpha(t)\rangle \mapsto e^{ik\Omega t}|\phi_\alpha(t)\rangle$. We shall compute the Floquet spectrum $\{\epsilon\}$ using the evolution operator $U(t)|\psi(0)\rangle = |\psi(t)\rangle$ and constructing the Floquet Hamiltonian $H_F = (i/T)\log[U(T)]$. Then, $H_F|\phi_\alpha(0)\rangle = \epsilon_\alpha|\phi_\alpha(0)\rangle$. The periodic eigenkets can be resolved in a Fourier series $|\phi(t)\rangle = \sum_k e^{-ik\Omega t}|\phi^{(k)}\rangle$. We shall use a shorthand $|\phi_\alpha\rangle$ for vectors in the extended Hilbert space spanned by $|\phi_\alpha^{(k)}\rangle$, with the inner product $\langle\phi'|\phi\rangle \equiv \sum_k \langle\phi'^{(k)}|\phi^{(k)}\rangle = \int_0^T \langle\phi'(t)|\phi(t)\rangle dt/T$ [6].

Floquet Majorana fermions.—Floquet Majorana fermions are bound states with quasienergy $\epsilon_0 = 0$ or $\epsilon_\pi = \Omega/2$ [3]. The particle-hole symmetry, $H_w^\dagger = -H_w$, requires $H_F^\dagger = -H_F$, so the quasienergies come in pairs $(\epsilon_\alpha, -\epsilon_\alpha)$. In the Nambu basis,

$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix} \phi_\alpha = \begin{pmatrix} u_\alpha \\ v_\alpha \end{pmatrix}$, the ϵ_0 and ϵ_π Floquet Majorana fermions $v_0(t) = u_0^*(t)$ and $v_\pi(t) = e^{i\Omega t}u_\pi^*(t)$.

In Fig. 1, we show the Floquet spectrum of a wire with a square-wave periodic chemical potential, $\mu(t)$, alternating with frequency Ω between μ_1 and μ_2 , respectively, over time intervals t_1 and t_2 in each period. Note that $\mu(t)$ is not in the topological range at any time. As Ω/W decreases, the Floquet band spreads and ϵ crosses ϵ_π , giving rise to a pair of ϵ_π Floquet Majorana fermion bound states. Additional gap-closing level crossings lead to the appearance or disappearance of ϵ_0 and ϵ_π Floquet Majorana fermions.

One expects that Floquet Majorana fermions appear for $\Omega \lesssim W$ where exchange processes with energy Ω become important and result in qualitative differences between the static energy and Floquet spectra [6, 44]. This is confirmed numerically in Fig. 1(a). It also agrees with a broad-band approximation when $\Omega \gg W$, using the Baker-Campbell-Hausdorff formula, $TH_F = t_1H_1 + t_2H_2 + t_1t_2[H_2, H_1] + \frac{1}{12}t_1t_2[t_2H_2 - t_1H_1, [H_2, H_1]] + \dots$. The first two terms yield a static Kitaev chain with an averaged chemical potential $\langle\mu\rangle = (t_1\mu_1 + t_2\mu_2)/T$. The only terms contributing to the commutator $[H_2, H_1]$ are $\Delta\sigma_x$ in $A_{r,r+1}$ and $-i\mu\sigma_y$ in $A_{r,r}$, yielding a σ_z term in $A_{r,r+1}$ that contributes to $\text{Im } w$. Physically, $\text{Im } w$ introduces a supercurrent in the chain, which renormalizes the spectral gap but, when small, leaves the topological phase boundary unchanged [38]. The same is true for the next term shown. We conclude that, when $2\langle|\mu|\rangle > W$, there are no Floquet Majorana fermions in the broad-band limit.

It is worth pointing out here that the quasienergy gap, ϵ_g , in the Floquet spectrum should not be naively used in comparison with other energy scales in the problem. For example, even when the average Hamiltonian $(t_1H_1 + t_2H_2)/T$ supports Majorana fermions, ϵ_g may be vanishingly small, as illustrated in Fig. 1(b) for the static limit $\mu_1 \rightarrow \mu_2$. In this case, the correct energy scale entering the adiabatic evolution is given by the actual energy gap. We shall see ϵ_g is relevant for the Floquet sum rule, though not for the conductance itself.

Transport.—Electrons in a driven system do not follow the usual statistics in a closed system. In the transport problem we can address this issue by assuming that the leads are static and follow the usual Fermi-Dirac statistics. The scattering problem between the leads can then be formulated by integrating out the leads using their Green's function [45, 46]. This procedure adds to H_w an imaginary self-energy $i\Gamma(t) = i\sum_\lambda \Gamma^\lambda(t)$, where $\Gamma^\lambda(t) = 2\text{Im} [K^{\lambda\dagger}g^\lambda(t)K^\lambda]$, and g^λ is the Green's function of lead λ . The wire's Green's function is periodic, $G(t+T, t'+T) = G(t, t')$, and satisfies

$$[\partial_t - A(t)]G(t, t') - (\Gamma * G)(t, t') = -i\delta(t - t'), \quad (3)$$

where $\Gamma * G$ is the convolution $\int_{-\infty}^t \Gamma(t-s)G(s, t')ds$.

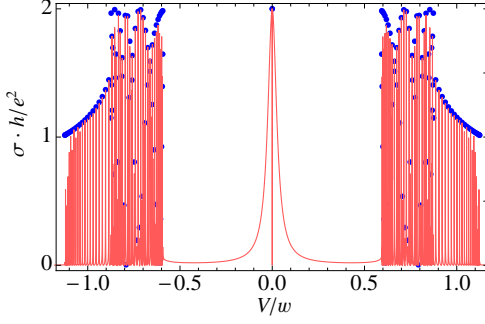


FIG. 2. Differential conductance $\sigma = dI/dV$ vs. bias V in a single-terminal setup for a static system with $L = 80$, $\Delta/w = 0.6$, $\mu/w = 0.25$ and $\nu/w = \pi/50$. The (blue) dots are calculated from the analytical expression of peak heights.

Then the steady state (time-averaged) current in lead λ , $J^\lambda = ie \langle [H(t), a^{\lambda\dagger} a^\lambda] \rangle$, can be computed with the Green's function. Assuming the leads' density of states ρ^λ is constant over the scattering energy range, we find an energy-independent $\Gamma^\lambda = -(\xi^\lambda + \xi^{\lambda\dagger})/2$, $\xi^\lambda = 2\pi K^{\lambda\dagger} \rho^\lambda K^\lambda$ and the differential conductance $\sigma^\lambda = dJ^\lambda/dV_\lambda$, with bias V_λ , reads

$$\sigma^\lambda = -\frac{e^2}{2\pi} \int d\omega \left[\sum_n \left\{ \text{Tr}[\xi^{\lambda\dagger} G^{(n)}(\omega) \xi^\lambda G^{(n)\dagger}(\omega)] f'_\lambda(\omega) + \text{Tr}[\xi^\lambda G^{(n)}(\omega) \xi^{\lambda\dagger} G^{(n)\dagger}(\omega)] f'_\lambda(-\omega) \right\} + C_\lambda(\omega) f'_\lambda(\omega) \right] \quad (4)$$

where $f_\lambda(\omega) = [1 + e^{(\omega - eV_\lambda)/\tau_\lambda}]^{-1}$ is the Fermi-Dirac distribution, $G^{(n)}(\omega) = \frac{1}{T} \int_0^T \int e^{in\Omega t} e^{i\omega s} G(t, t-s) ds dt$, and $C_\lambda(\omega) = \sum_{\kappa \neq \lambda, n} \text{Tr}[(\xi^\kappa + \xi^{\kappa\dagger}) G^{(n)}(\omega) \xi^\lambda G^{(n)\dagger}(\omega)]$ [46]. The static case is found by setting $G^{(n)}(\omega) = \delta_{n,0} G(\omega)$.

For a single lead with a point contact, C vanishes identically and the contact matrix is zero except at the contact site where $\xi = \nu(1 - \sigma_y)/2$ with $\nu = 2\pi\rho|w|^2$. Then,

$$\sigma = -\frac{e^2\nu^2}{2\pi} \sum_n \int \left[|\mathcal{G}_{\text{he}}^{(n)}(\omega)|^2 + |\mathcal{G}_{\text{eh}}^{(n)}(-\omega)|^2 \right] f'(\omega) d\omega, \quad (5)$$

where the \mathcal{G}_{eh} and \mathcal{G}_{he} are the off-diagonal elements of the Nambu-Gorkov Green's function at the contact site,

$$\mathcal{G}^{(n)}(\omega) = \sum_{\alpha k} \frac{|\varphi_\alpha^{(n+k)}\rangle \langle \bar{\varphi}_\alpha^{(k)}|}{\omega - \epsilon_\alpha - n\Omega + i\delta_\alpha}, \quad (6)$$

with $-i\delta_\alpha$ the self-energy correction to quasienergy ϵ_α , and $|\varphi_\alpha\rangle$ and $\langle \bar{\varphi}_\alpha|$, respectively, the right and left Floquet eigenvectors of the effective (non-Hermitian) Hamiltonian $H_w + i\Gamma$ at level α .

In the weak-contact limit $\nu/w \ll 1$, we can employ perturbation theory in Γ . To the leading order, we find $|\varphi_\alpha\rangle = |\phi_\alpha\rangle$, $\langle \bar{\varphi}_\alpha| = \langle \phi_\alpha|$ (i.e. the same as eigenvectors of H_w), and $\delta_\alpha = -\langle \phi_\alpha | \Gamma | \phi_\alpha \rangle$ [6]. Let us first work

out the static case. Then, $\delta_\alpha = \frac{1}{2}\nu(|u_\alpha^c|^2 + |v_\alpha^c|^2)$ with u_α^c and v_α^c evaluated at the contact site. At zero temperature, $\lim_{V \rightarrow E_\alpha} \sigma(V) = \sigma_\alpha L(\frac{V-E_\alpha}{\delta_\alpha})$ where E_α is an energy level of the static system, $L(z) = (1+z^2)^{-1}$ is the Lorentzian, and the peak value,

$$\sigma_\alpha = \frac{2e^2}{2\pi} \left| \frac{2u_\alpha^c v_\alpha^c}{|u_\alpha^c|^2 + |v_\alpha^c|^2} \right|^2. \quad (7)$$

For the zero-energy Majorana fermion, $u_0 = v_0^*$, so $\sigma_0 = 2e^2/h$ in restored units, as is well known [23, 47]. In Fig. 2, we compare this analytical expression with a full numerical solution.

Floquet sum rule.—In the driven system, the peaks at $V = \epsilon_0$ and $V = \epsilon_\pi$ are not quantized even when Floquet Majorana fermions are present. This is because energies $\epsilon_\alpha + n\Omega$ are all connected via the drive force by emission and absorption processes. Instead, we find a “Floquet sum rule” for the sum of differential conductance at these energies [46],

$$\tilde{\sigma}(V) = \sum_n \sigma(V + n\Omega). \quad (8)$$

At zero temperature, $\lim_{V \rightarrow \epsilon_\alpha} \tilde{\sigma}(V) = \tilde{\sigma}_\alpha L(\frac{V-\epsilon_\alpha}{\delta_\alpha})$ is, again, a Lorentzian with the peak value,

$$\tilde{\sigma}_\alpha = \frac{2e^2}{2\pi} \left| \frac{2\|u_\alpha^c\| \|v_\alpha^c\|}{\|u_\alpha^c\|^2 + \|v_\alpha^c\|^2} \right|^2, \quad (9)$$

where $\|z\|^2 = \sum_k |z^{(k)}|^2$. By particle-hole symmetry $\|u_0\| = \|v_0\|$ and $\|u_\pi\| = \|v_\pi\|$. Thus, if there is a Floquet Majorana fermion at ϵ_0 and/or ϵ_π ,

$$\tilde{\sigma}_0 \equiv \tilde{\sigma}(\epsilon_0) = \frac{2e^2}{h} \quad \text{and/or} \quad \tilde{\sigma}_\pi \equiv \tilde{\sigma}(\epsilon_\pi) = \frac{2e^2}{h}, \quad (10)$$

respectively. These relations can be generalized for non-point contact terms as well. This is our central result.

The two Floquet Majorana fermions overlap and split away from ϵ_0 or ϵ_π by an amount λ that is exponentially small in their separation. When $\lambda > \nu$, $\tilde{\sigma}$ also splits with the peak values $\tilde{\sigma}_0$ and $\tilde{\sigma}_\pi$ shifting to $V = \epsilon_0 \pm \lambda$ and $V = \epsilon_\pi \pm \lambda$, respectively, each with half the widths of the central peak. Therefore, the total weight stays the same. This is a general feature: the total weight $\int_0^\Omega \tilde{\sigma}(V) dV = \int_{-\infty}^\infty \sigma(V) dV \propto \sum_n \int |\mathcal{G}_{\text{eh}}^{(n)}(\omega)|^2 d\omega$ is constant at all temperatures.

We have numerically investigated the Floquet sum-rule quantization in the quantum wire. The plots in Fig. 3 show the steady differential conductance calculated for a two-lead setup with symmetric biases $\pm V$. It is clear that, within our numerical precision, $\tilde{\sigma}_0$ and/or $\tilde{\sigma}_\pi$ are quantized at $2e^2/h$ exactly when ϵ_0 and/or ϵ_π Floquet Majorana fermions appear. Note that the individual peaks of σ at $V = n\Omega$ (for ϵ_0 Floquet Majorana fermion) or $V = (2n+1)\Omega/2$ (for ϵ_π Floquet Majorana fermion)

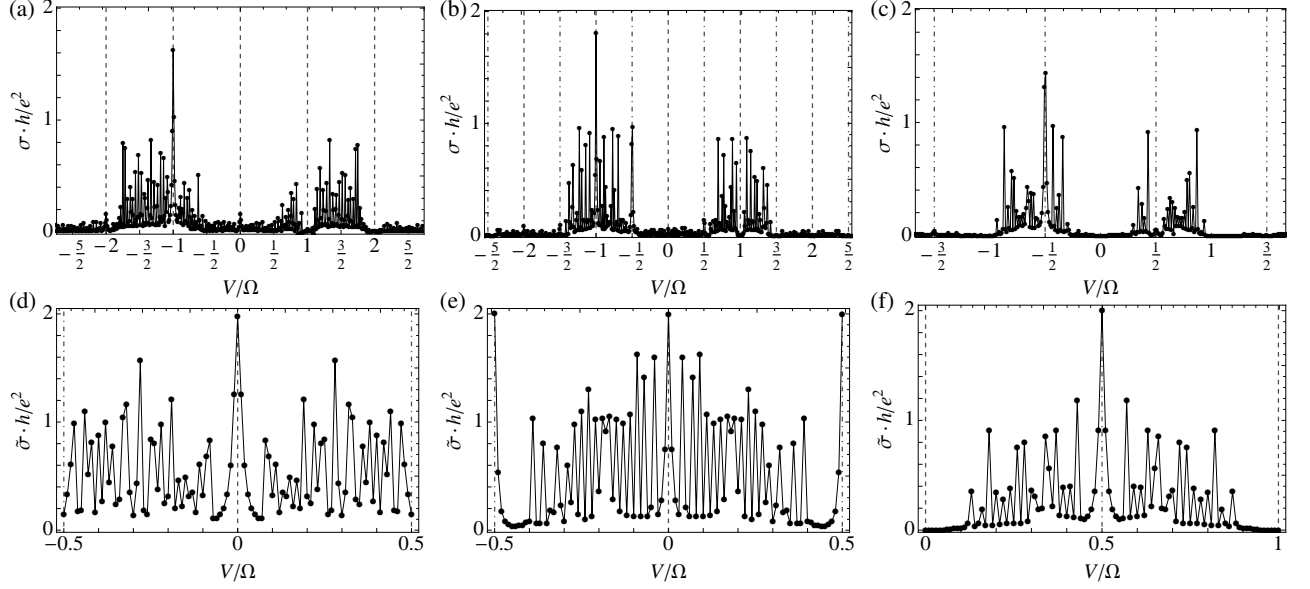


FIG. 3. Differential conductances σ (top row) and $\tilde{\sigma}$ (bottom row) of the driven system as a function of bias V/Ω in a two-terminal setup. The parameters Δ, μ_1, μ_2 and t_1/t_2 are as in Fig. 1(a), $\nu/w = \pi/25$, and the other parameters are: (a,d) $L = 40, \Omega/W = 0.37$, (b,e) $L = 70, \Omega/W = 0.49$, (c,f) $L = 40, \Omega/W = 0.75$. The frequencies are marked by arrows in Fig. 1(a).

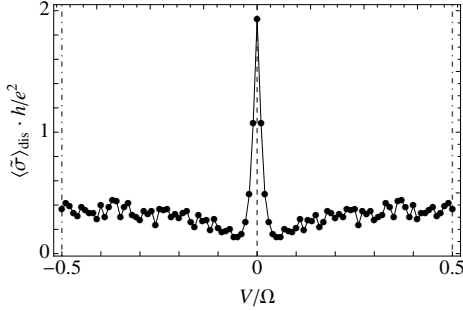


FIG. 4. Differential conductance in a two-terminal setup averaged over 50 disorder configurations. The parameters are as in Fig. 3 (a,d) and disorder strength $\mu_d = 0.07W = 0.19\Omega$.

are not quantized. Indeed, the main contribution is not even from $n = 0$ [46]. The Floquet spectrum is naturally reflected in $\tilde{\sigma}$: The quantized peaks at ϵ_0 (or ϵ_π) are separated from the other peaks by a value of $V \sim \epsilon_{g,0}$ (or $\epsilon_{g,\pi}$), i.e. the gap in the quasienergy gap separating the respective Floquet Majorana fermions from the other states. The quasienergy gaps in Fig. 3 are $\sim 0.1\Omega$ except for $\epsilon_{g,0} \sim 0.05\Omega$ in Fig. 3(e).

Effects of disorder.—The natural question to answer at this point is whether and how $\tilde{\sigma}$ could be measured in an actual experiment. Especially important is to be able to tell apart a quantized peak from the other features, which is complicated if $\epsilon_{g,0}$ and $\epsilon_{g,\pi}$ are small. A possible way around is to exploit the topological character of the quantization of $\tilde{\sigma}_0$ and $\tilde{\sigma}_\pi$. Specifically, they must be protected against disorder while the other features

are not. We have studied the effects of disorder numerically by adding a static, uncorrelated, random $\delta\mu_r$ to the wire's chemical potential at site r , i.e. $\langle\delta\mu_r\rangle_{\text{dis}} = 0$ and $\langle\delta\mu_r\delta\mu_{r'}\rangle_{\text{dis}} = \mu_d^2\delta_{rr'}$ where μ_d is the disorder strength. A typical result for the disorder-averaged $\langle\tilde{\sigma}\rangle_{\text{dis}}$ at moderate disorder is shown in Fig. 4. The central quantized peak remains nearly unchanged, while the other peaks are suppressed significantly. Note that here $\mu_d > \epsilon_{g,0}$. For stronger disorder the quantized peak is suppressed as well [46].

Concluding remarks.—In sum, we find that the differential conductance summed over periodic drive harmonics, $\tilde{\sigma}$, signals Floquet Majorana fermions with a topologically protected quantized value $2e^2/h$ at the Floquet Majorana quasienergy. The quantization is robust and most prominent in the presence of weak disorder. This suggests disorder can be used as a knob to probe Floquet Majorana fermions.

We have studied the finite temperature (τ) behavior of $\tilde{\sigma}$ and found [46] that for temperature $\nu \ll \tau \lesssim \epsilon_g$, the Majorana peak height falls as $\tilde{\sigma}_{0,\pi} \sim 1/\tau$. For $\tau \gtrsim \epsilon_g$, the non-topological peaks start to overlap with the Majorana peak and $\tilde{\sigma}_{0,\pi}$ saturates.

In closing, we sketch some problems for future studies. Other transport signatures of Floquet Majorana fermions, such as noise and heat transport, are interesting, open problems. Our inclusion of disorder was only as a static component of the chemical potential. This is appropriate if disorder potential is intrinsic to the wire itself and is not affected by the external drive. Other disorder configurations, e.g. in the contacts or the external drive

itself, would be interesting to study in future. In particular, disorder in the drive spoils the strict periodicity and Floquet structure. Quantifying the effects of disorder is another interesting problem: the usual way of relating the disorder mean-free path to the density of states in the static system does not work here, since the periodic chemical potential is never in the topological band, so the relevant density of states vanishes. Finally, the effects of disorder at finite temperature as well as interactions are left to future studies.

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SUPPLEMENTAL MATERIAL

Here we sketch the details for the derivation of the conductance and *Floquet sum rule* described in the main text. We employ Green's function approach for deriving the charge current in the system described by time dependent Hamiltonian $H_w(t)$ and contact Hamiltonian H_c , as discussed in the main text

$$H_w(t) = \frac{i}{2} \gamma^\dagger A(t) \gamma, \quad H_c = \sum_\lambda a^{\lambda\dagger} K^\lambda \gamma + \text{h.c.} \quad (\text{S1})$$

The net charge current flowing across the contact λ into the wire is ($\hbar = 1$)

$$\begin{aligned} J^\lambda(t) &= ie [H_w(t) + H_c, N^\lambda(t)] \\ &= ie (\gamma^\dagger(t) K^{\lambda\dagger} a^\lambda(t) - \text{h.c.}), \end{aligned} \quad (\text{S2})$$

where N^λ is the number operator for electrons in lead λ . One can use the solution of Heisenberg' equation for the electron (in the lead) $a^\lambda(t)$

$$a^\lambda(t) = \eta^\lambda(t) + \int_{t_0 \rightarrow -\infty}^t g^\lambda(t-t') K^\lambda \gamma(t') dt', \quad (\text{S3})$$

where, t_0 is the *switching time*, $g^\lambda(t-t')$ is the Green's function matrix of electrons in lead λ . In the wide band limit the density of states ρ^λ of lead λ is constant for the relevant energy scales and in the simplest situation $g^\lambda(\omega) = -i\pi\rho^\lambda$. The *noise* term $\eta^\lambda(t) = ig^\lambda(t-t_0)a^\lambda(t_0)$ obeys the fluctuation-dissipation relation after averaging

$$J^\lambda = \frac{e}{2\pi} \sum_{\lambda'} \int d\omega \left\{ \text{Tr} \left[\xi^\lambda G(\omega) \xi^{\lambda'} G^\dagger(\omega) \right] [f_{\lambda'}(\omega) - f_\lambda(\omega)] + \text{Tr} \left[\xi^\lambda G(\omega) \xi^{\lambda'\dagger} G^\dagger(\omega) \right] [1 - f_{\lambda'}(-\omega) - f_\lambda(\omega)] \right\}, \quad (\text{S10})$$

where $\xi^\lambda = 2\pi K^{\lambda\dagger} \rho^\lambda K^\lambda$ and $J^\lambda = \int \langle J^\lambda(t) \rangle dt$. The conductance is obtained by taking the derivative with respect to the bias V_λ ,

$$\begin{aligned} \sigma^\lambda &= -\frac{e^2}{2\pi} \int d\omega \text{Tr} \left[\xi^\lambda G(\omega) \xi^{\lambda\dagger} G^\dagger(\omega) \right] [f'_\lambda(-\omega) + f'_\lambda(\omega)] \\ &\quad - \frac{e^2}{2\pi} \sum_{\lambda' \neq \lambda} \int d\omega \text{Tr} \left[\xi^\lambda G(\omega) (\xi^{\lambda'} + \xi^{\lambda'\dagger}) G^\dagger(\omega) \right] f'_\lambda(\omega). \end{aligned} \quad (\text{S11})$$

over the lead states

$$\langle \eta_r^{\lambda\dagger}(\omega) \eta_{r'}^{\lambda'}(\omega') \rangle = (2\pi)^2 \delta_{\lambda\lambda'} \rho_{rr'}^\lambda f_\lambda(\omega) \delta(\omega - \omega'), \quad (\text{S4})$$

$$\langle \eta_r^\lambda(\omega) \eta_{r'}^{\lambda'\dagger}(\omega') \rangle = (2\pi)^2 \delta_{\lambda\lambda'} \rho_{rr'}^\lambda \bar{f}_\lambda(\omega) \delta(\omega - \omega'), \quad (\text{S5})$$

where $f_\lambda(\omega) = 1 - \bar{f}_\lambda(\omega) = [1 + e^{(\omega - eV_\lambda)/\tau_\lambda}]^{-1}$ is the Fermi-Dirac distribution of the lead λ with bias V_λ and temperature τ_λ . (The Boltzmann constant $k_B = 1$.) For the Majorana operator $\gamma(t)$, the integration of the Heisenberg equation can be complicated by the time dependence of $A(t)$. In a periodically driven system it is obtained in terms of the Floquet Green's function, as we discuss later.

Static System

If the time dependence of $A(t)$ is trivial, one can integrate the Heisenberg's equation for the Majorana operator with using Eq. (S3). In the Fourier space,

$$\gamma(\omega) = G(\omega) h(\omega), \quad (\text{S6})$$

where $G(\omega)$ is the Green's function defined from

$$G^{-1}(\omega) = \omega - iA - i\Gamma(\omega), \quad (\text{S7})$$

with the self energy

$$i\Gamma(\omega) = \left[K^{\lambda\dagger} g^\lambda(\omega) K^\lambda - K^{\lambda\dagger} g^{\lambda*}(-\omega) K^{\lambda*} \right] \quad (\text{S8})$$

and

$$h(\omega) = \sum_\lambda \left[K^{\lambda\dagger} \eta^\lambda(\omega) - K^{\lambda\dagger} \eta^{\lambda*}(-\omega) \right]. \quad (\text{S9})$$

Using Eq. (S6) and Eq. (S3) in the current equation Eq. (S2) and averaging over the lead states one obtains [1]

The second term vanishes if the system has a single lead,

$$\begin{aligned} \sigma^\lambda &= -\frac{e^2}{2\pi} \int d\omega \text{Tr} \left[\xi^\lambda G(\omega) \xi^{\lambda\dagger} G^\dagger(\omega) \right] \\ &\quad \times [f'_\lambda(-\omega) + f'_\lambda(\omega)]. \end{aligned} \quad (\text{S12})$$

Periodically driven system

In a driven system, the formulation of current by non-equilibrium Floquet Green's function has been discussed

before [2–4] and we follow the same strategy. Integrating the Heisenberg equation for the Majorana operator in a driven system gives [5]

$$\left[i \frac{\partial}{\partial t} - iA(t) \right] \gamma(t) - i \int_0^\infty ds \Gamma(s) \gamma(t-s) = h(t), \quad (\text{S13})$$

with self energy $i\Gamma(s) = 2i \sum_\lambda \text{Im} [K^{\lambda\dagger} g^\lambda(s) K^\lambda]$ and $h(t) = 2i \sum_\lambda \text{Im} [K^{\lambda\dagger} \eta^\lambda(t)]$. The Green's function of this inhomogeneous equation $G(t, t')$ satisfies

$$\left[i \frac{\partial}{\partial t} - iA(t) \right] G(t, t') - i \int_0^\infty ds \Gamma(s) G(t-s, t') = \delta(t-t'). \quad (\text{S14})$$

For a periodic drive with period $T = 2\pi/\Omega$, the *Floquet Green's function* is also periodic over the same period

$G(t+T, t'+T) = G(t, t')$. One can introduce the Fourier transform,

$$G(t, t') = \sum_k \int \frac{d\omega}{2\pi} G^{(k)}(\omega) e^{-i\omega(t-t')} e^{-ik\Omega t}, \quad (\text{S15})$$

The Majorana operator is solved in terms of the Green's function

$$\gamma(t) = \sum_k \int \frac{d\omega}{2\pi} e^{-i\omega t} e^{-ik\Omega t} G^{(k)}(\omega) h(\omega). \quad (\text{S16})$$

Using these expressions in the current formula Eq. (S2), the the steady state current $J^\lambda = \langle\langle J^\lambda \rangle\rangle = \int_0^T \langle J^\lambda(t) \rangle dt / T$ in the wide band limit is found to be

$$J^\lambda = -\frac{e}{2\pi} \sum_{\lambda'} \int d\omega \sum_l \left\{ \text{Tr} \left[G^{(l)\dagger}(\omega) (\xi^{\lambda'} + \xi^{\lambda'\dagger}) G^{(l)}(\omega) \xi^\lambda \right] f_\lambda(\omega) \right. \\ \left. - \text{Tr} \left[\xi^\lambda G^{(l)}(\omega) \xi^{\lambda'} G^{(l)\dagger}(\omega) \right] f_{\lambda'}(\omega) - \text{Tr} \left[\xi^\lambda G^{(l)}(\omega) \xi^{\lambda'\dagger} G^{(l)\dagger}(\omega) \right] [1 - f_{\lambda'}(-\omega)] \right\}, \quad (\text{S17})$$

which gives the conductance

$$\sigma^\lambda = -\frac{e^2}{2\pi} \int d\omega \sum_l \text{Tr} \left[\xi^\lambda G^{(l)}(\omega) \xi^{\lambda\dagger} G^{(l)\dagger}(\omega) \right] [f'_\lambda(-\omega) + f'_\lambda(\omega)] \\ - \frac{e^2}{2\pi} \sum_{\lambda' \neq \lambda} \int d\omega \sum_l \text{Tr} \left[\xi^\lambda G^{(l)}(\omega) (\xi^{\lambda'} + \xi^{\lambda'\dagger}) G^{(l)\dagger}(\omega) \right] f'_\lambda(\omega). \quad (\text{S18})$$

Finally, similar to the static problem, for a single lead,

$$\sigma^\lambda = -\frac{e^2}{2\pi} \sum_l \int d\omega \text{Tr} \left[\xi^\lambda G^{(l)}(\omega) \xi^{\lambda\dagger} G^{(l)\dagger}(\omega) \right] \\ \times [f'_\lambda(-\omega) + f'_\lambda(\omega)]. \quad (\text{S19})$$

Relation to Nambu-Gorkov Green's function

The Majorana representation is related to the Nambu spinor $(c_r, c_r^\dagger) = \psi_r^\dagger$ by the unitary transformation $\gamma_r = u\psi_r$ with $u = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}$ for each site r . And the Bogoliubov-de Gennes (BdG) Hamiltonian is related as $H_{\text{BdG}} = iU^\dagger A U$, where $U = u \oplus u \oplus \dots \oplus u$ (L times). The Nambu-Gorkov Green's function in a static system is defined as

$$\mathcal{G}(\omega) = (\omega - H_{\text{BdG}} + i\epsilon)^{-1}, \quad (\text{S20})$$

with vanishing positive ϵ .

For a single lead with point contact, the matrix $\xi = \nu(1 - \sigma_y)/2$. By unitary transforming the Eq. (S12) with U we get

$$\sigma = -\frac{e^2 \nu^2}{2\pi} \int d\omega |\mathcal{G}_{\text{eh}}(\omega)|^2 [f'_\lambda(-\omega) + f'_\lambda(\omega)], \quad (\text{S21})$$

where \mathcal{G}_{eh} is the electron-hole component of the Green's function evaluated at the contact site

$$\mathcal{G}_c(\omega) = \begin{pmatrix} \mathcal{G}_{\text{ee}}(\omega) & \mathcal{G}_{\text{eh}}(\omega) \\ \mathcal{G}_{\text{he}}(\omega) & \mathcal{G}_{\text{hh}}(\omega) \end{pmatrix}, \quad (\text{S22})$$

For the periodically driven system the relevant Green's function is the Floquet Green's function in Nambu spinor basis, which is defined as [5]

$$\mathcal{G}^{(l)}(\omega) = \sum_{\alpha k} \frac{|\varphi_\alpha^{(l+k)}\rangle \langle \bar{\varphi}_\alpha^{(k)}|}{\omega - \epsilon_\alpha - l\Omega + i\delta_\alpha}, \quad (\text{S23})$$

where $|\varphi_\alpha^{(i)}\rangle$ and $\langle\bar{\varphi}_\alpha^{(k)}|$ are the discrete Fourier component of the time periodic function 'Floquet states' $|\varphi_\alpha(t)\rangle$ and $\langle\bar{\varphi}_\alpha(t)|$, which are right and left eigenstates of the effective Floquet Hamiltonian $H_{\text{eff}} = H_w + i\Gamma - i\frac{\partial}{\partial t}$ with eigenvalues (*Floquet energies*) $\epsilon_\alpha - i\delta_\alpha$.

$$\begin{aligned} H_{\text{eff}} |\varphi_\alpha(t)\rangle &= (\epsilon_\alpha - i\delta_\alpha) |\varphi_\alpha(t)\rangle, \\ \langle\bar{\varphi}_\alpha(t)| H_{\text{eff}} &= (\epsilon_\alpha - i\delta_\alpha) \langle\bar{\varphi}_\alpha(t)|. \end{aligned} \quad (\text{S24})$$

$|\varphi_\alpha(t)\rangle$ lives in an extended (with the time axis) Hilbert space [6], where the expectation of any operator \mathcal{O} in the state is defined with the time average

$$\langle\langle\mathcal{O}\rangle\rangle = \frac{1}{T} \int_0^T dt \langle\bar{\varphi}_\alpha(t)| \mathcal{O} |\varphi_\alpha(t)\rangle = \sum_k \langle\bar{\varphi}_\alpha^{(k)}| \mathcal{O} |\varphi_\alpha^{(k)}\rangle. \quad (\text{S25})$$

In terms of the Green's functions $\mathcal{G}^{(l)}$, one can express the conductance for a periodically driven system with single lead and contact matrix $\xi = \nu(1 - \sigma_y)/2$ similar to the static case,

$$\sigma = -\frac{e^2\nu^2}{2\pi} \sum_n \int \left[|\mathcal{G}_{\text{he}}^{(n)}(\omega)|^2 + |\mathcal{G}_{\text{eh}}^{(n)}(-\omega)|^2 \right] f'(\omega) d\omega, \quad (\text{S26})$$

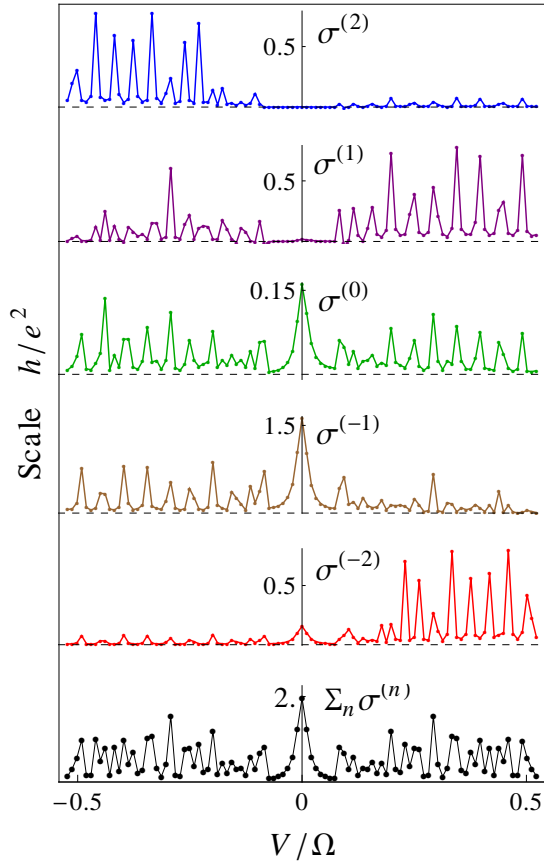


FIG. S1. Contributions to Floquet sum rule from different n in $\tilde{\sigma}(V) = \sum_n \sigma(n\Omega + V)$. Here $\sigma^{(n)} = \sigma(n\Omega + V)$. The parameters are as in Fig. 3 (c,f) of the main text.

Now, the eigenstate of the time dependent Hamiltonian H_w , $|\psi_\alpha(t)\rangle$ are related to the Floquet state $|\varphi_\alpha(t)\rangle$ by the relation $|\psi_\alpha(t)\rangle = e^{-i(\epsilon_\alpha - i\delta_\alpha)t} |\varphi_\alpha(t)\rangle$. For 0 and π/T Floquet energy, $|\psi_0(t)\rangle = e^{-\delta_0 t} |\varphi_0(t)\rangle$ and $|\psi_\pi(t)\rangle = e^{-i\pi t/T} e^{-\delta_\pi t} |\varphi_\pi(t)\rangle$. Considering the contact term perturbatively, in the leading order one can neglect the modification to the wavefunction to have $|\psi_0(t)\rangle \approx |\varphi_0(t)\rangle$ and $|\psi_\pi(t)\rangle \approx e^{-i\pi t/T} |\varphi_\pi(t)\rangle$. Using the particle-hole symmetry at Floquet energies 0, π/T , one can express the eigenstates in the Bogoliubov quasiparticle representation $|\psi_0(t)\rangle = (u_0(t), u_0^*(t))^\top$ and $|\psi_\pi(t)\rangle = (u_\pi(t), u_\pi^*(t))^\top$. This allows us to write $|\varphi_i(t)\rangle$ in the Fourier space

$$|\varphi_0^{(l)}\rangle = \begin{pmatrix} u_0^{(l)} \\ u_0^{(-l)*} \end{pmatrix}, \quad |\varphi_\pi^{(l)}\rangle = \begin{pmatrix} u_0^{(l)} \\ u_0^{(-l+1)*} \end{pmatrix}. \quad (\text{S27})$$

The leading perturbation to the self-energy part can be computed in the extended Hilbert space [6] with

$$\Gamma = -(\xi + \xi^\top)/2 = -\frac{\nu}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}_c \oplus \underbrace{0 \oplus 0 \dots \oplus 0}_{L-1 \text{ times}}, \quad (\text{S28})$$

$$\delta_\alpha = -\langle\langle\varphi_\alpha(t)| \Gamma |\varphi_\alpha(t)\rangle\rangle = \frac{\nu}{2} (\|u_\alpha^c\|^2 + \|v_\alpha^c\|^2), \quad (\text{S29})$$

where $u_\alpha^{c(k)}$ and $v_\alpha^{c(k)}$ are evaluated at the contact site and $\|z\|^2 = \sum_k |z^{(k)}|^2$. We also note that

$$\begin{aligned} \lim_{V \rightarrow \epsilon_\alpha} \sum_k |\mathcal{G}_{\text{eh}}^{(l)}(V + k\Omega)|^2 &\approx \sum_{lk} \left| \frac{u_\alpha^{c(k+l)} v_\alpha^{c(k)}}{V - \epsilon_\alpha + i\delta_\alpha} \right|^2 \\ &= \frac{4}{|\nu|^2} \left| \frac{\|u_\alpha^c\| \|v_\alpha^c\|}{\|u_\alpha^c\|^2 + \|v_\alpha^c\|^2} \right|^2 L\left(\frac{V - \epsilon_\alpha}{\delta_\alpha}\right), \end{aligned} \quad (\text{S30})$$

where $L(z) = (1 + z^2)^{-1}$ is the Lorentzian. The peak value at zero temperature is

$$\tilde{\sigma}_\alpha = \lim_{V \rightarrow \epsilon_\alpha} \sum_k \sigma(V + k\Omega) = \frac{2e^2}{2\pi} \left| \frac{2\|u_\alpha^c\| \|v_\alpha^c\|}{\|u_\alpha^c\|^2 + \|v_\alpha^c\|^2} \right|^2. \quad (\text{S31})$$

This is our Eq. (9) in the main text.

Numerical Results

In Fig. S1 we plot $\tilde{\sigma}(V) = \sum_n \sigma(V + n\Omega)$ for system with two symmetrically biased leads ($V_1 = -V_2 = -V$) hosting a ϵ_0 Floquet Majorana fermion, which shows that the peaks of individual components $\sigma(V + n\Omega)$ are not quantized, but the sum $\tilde{\sigma}$ shows a quantized peak.

In Fig. S2, we compare the differential conductance $\tilde{\sigma}$ for smaller and larger than optimal disorder strength. For small disorder, the non-topological peaks do not get

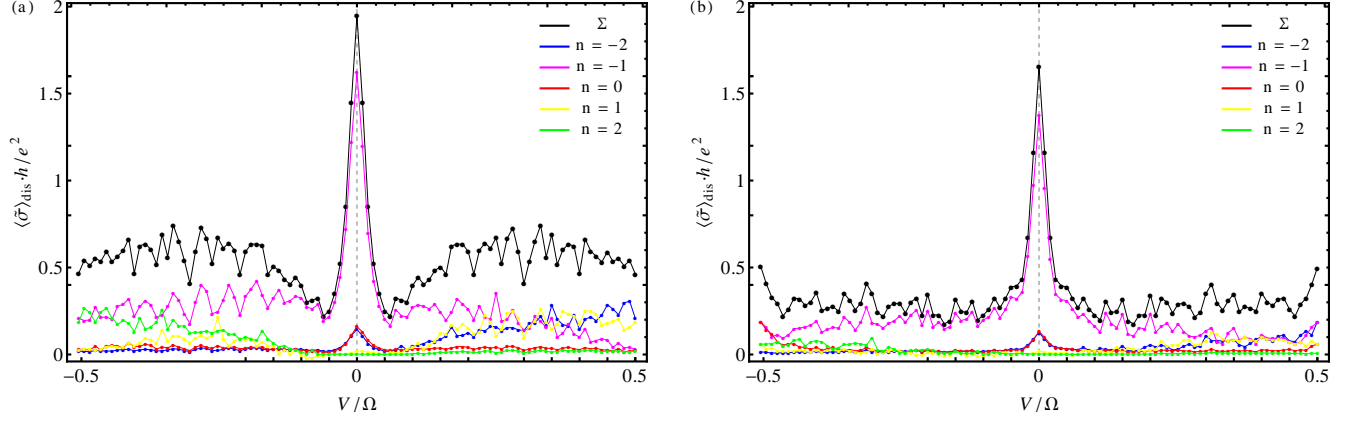


FIG. S2. Differential conductance in a two-terminal setup averaged over 50 disorder configurations, where disorder strengths (a) $\mu_d = 0.05W = 0.13\Omega$ and (b) $\mu_d = 0.11W = 0.30\Omega$ which are smaller and larger than the disorder strength used in the main text. Other parameters are as in Fig. S1, except here $\nu/w = \pi/17$. We show the contribution to the sum, $\tilde{\sigma}(V) \equiv \Sigma$, from different $\sigma^{(n)}(V) = \sigma(V + n\Omega)$.

suppressed enough compared to the topological peak, whereas for large disorder the topological peak can get suppressed.

Finally, in Fig. S3 we plot the temperature dependence of $\tilde{\sigma}(0)$ with temperature (τ). As τ becomes comparable to the Floquet gap ϵ_g , the peak becomes flat.

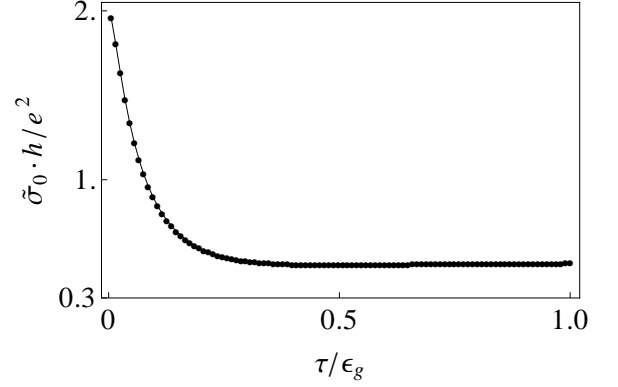


FIG. S3. Temperature (τ) dependence of the peak height of conductance $\tilde{\sigma}(0) \equiv \sigma_0$ for Floquet Majorana. The parameters used are same as that of Fig. S1. The Floquet gap is $\epsilon_g = 0.15\Omega$.

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